Macro Competition 2016

Proposed Solution as PID Controller

We follow notation and main derivations from wiki\PID_controller. In our case, the control variable u we want to exert influence on is the NACA profile angle. We want this influence to be a function of the lift. More specifically, to be a function of the error

$$e = (\text{ LiftGoal} - \text{CurrentLift }).$$

We could choose the most general form for a PID controller

$$u(t) = C + K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t) \tag{1}$$

and use this FAQ How to get time integral and time derivative to implement the integral and derivative terms in STAR-CCM+.

Nonetheless, we do not need to follow that path. The key point to observe is that we are always working with discretized approximations to derivatives and integrals. More specifically, lets consider the discretized version of equation (1)

$$u(t_n) = C + K_p e(t_n) + K_i \sum_{j=0}^{n} e(t_j) \Delta t + K_d \frac{e(t_n) - e(t_{n-1})}{\Delta t}$$

where t_n could refer either to the n-th iteration or n-th time step. Clearly, at time/iteration $n-1$ you have

$$u(t_{n-1}) = C + K_p e(t_{n-1}) + K_i \sum_{j=0}^{n-1} e(t_j) \Delta t + K_d \frac{e(t_{n-1}) - e(t_{n-2})}{\Delta t}.$$

After substracting you will obtain

$$u(t_n) - u(t_{n-1}) = K_p(e(t_n) - e(t_{n-1})) + K_i e(t_n) \Delta t + \frac{K_d}{\Delta t}(e(t_n) - 2e(t_{n-1}) + e(t_{n-2})).$$

Reordering terms you will get

$$u(t_n) = u(t_{n-1}) + c_0 e(t_n) + c_1 e(t_{n-1}) + c_2 e(t_{n-2})$$

where

$$\begin{aligned} c_0 &= K_p + K_i \Delta t + \frac{K_d}{\Delta t} \\ c_1 &= -\big(K_p - \frac{2K_d}{\Delta t}\big) \\ c_2 &= \frac{K_d}{\Delta t} \end{aligned}$$

For the proposed solution we have $\Delta t = 1, c_1 = 0, c_2 = 0$ and the following equivalences:

$$
\begin{aligned}
u(t_n) &\rightarrow \text{newAngle} \\
u(t_{n-1}) &\rightarrow \text{currentAngle} \\
e(t_n) &\rightarrow -(\text{Lift - LiftGoal}) \\
c_0 = K_i &\rightarrow \text{couplingCoeff} = 0.001.
\end{aligned}
$$

Thus our implementation corresponds to a pure integral controller.

If you want to implement the more general PID controller then you will merely need to have access to past values $e(t_{n-1})$ and $e(t_{n-2})$. This can be done with field mean monitors as shown in How to store and access results from previous iteration or previous time step?. Please follow also the links within this Steve portal article for more information.